Grade Inflation, Social Background, and Labour Market Matching[a]

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Abstract

A model is presented where workers of differing abilities and from different social backgrounds are assigned to jobs based on grades received at school. It is examined how this matching is affected if good grades are granted to some low-ability students. Such grade inflation is shown to reduce the aggregate wage of the lower class workers because employers use social origin as a signal for productivity if grades are less than fully informative. Moreover, the high-ability students from the higher class may benefit from grade inflation since this shields them from the competition on the part of able students from the lower classes.

Keywords: education, grading, standards, assignment, social mobility
JEL: I21, J24, C78

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1 Introduction

For many students, examinations are the most important aspect of schooling, and many of them feel substantial pressure to improve the grades they obtain in such examinations. This attitude is most pronounced when examination results have immediate and important consequences for the student such as failure to graduate from high school, as is the case under the policy of “high stakes testing” in many U.S. states. Because of this impact on students’ well-being, some observers have reservations about tough examination and grading standards. For example, the California Teachers Association suggests to complement the current California High School Exit Exam by “parallel forms of assessment” such as “essays and personal communications” (see California Teachers Association, 2006). This should allow to reduce the rate of students failing in the assessment which is considered to be excessive. In addition to the general impact of tough standards on all students, it is specifically feared that students from disadvantaged backgrounds are hurt disproportionately by tough examination standards. This concern is rooted in the fact that, on average, students from poor, minority, or immigrant families perform less well in tests (see Madaus and Clarke, 2001). As a consequence, one might think that softening the grading scheme or lowering graduation standards will improve the opportunities of these students. While these considerations suggest that examination standards are too tough, the opposite view is also expressed. Proponents of this view complain that good grades are awarded too easily so that employers and further education institutions cannot count on the qualification of a candidate with a good grade. Specifically, there seems to be widespread belief that such a practice, which is labelled “grade inflation”, occurs routinely at the university level, even at the most prestigious institutions (see for example Mansfield, 2001).

Addressing this debate, the present paper examines the relationship between grades, social origin, and the labour market. It serves two purposes. First, on a rather fundamental level, a model is presented which allows to analyse in a natural way the role of grades for the allocation on the labour market. This model is a continuous version of the two-sided matching model introduced by Roth and Sotomayor (1990), and later used, for example, by Cole, Mailath, and Postlewaite (1992), Corneo and Grüner (2000), and Clark and Kanbur (2004) in the context of social interaction. Following these contributions, the concept of a stable assignment is used to describe the allocation on the labour market. In a stable assignment, the matching between firms and workers and the resulting payoffs are such that no agent strictly prefers to stay alone, and such that no pair consisting of a firm and a worker can improve on their payoffs by matching together.

In the model presented here, there are workers of high and low ability. The central feature of the model is that workers with high ability, while being more productive than low-ability workers on every job, have a comparative advantage on the most demanding jobs. Thus, the labour market should match the most able workers with the most productive jobs and allocate the less demanding jobs to workers with lower ability. In such a context, grades serve an obvious information function by signalling ability to firms. To reflect the concern about the informational content of grades mentioned above, a very simple form of grade inflation is introduced which consists of awarding a good grade (an "A") to a fraction of low-ability students. The matching of workers to firms and the resulting wages in a stable assignment are characterised as a function of the extent of grade inflation. It is shown that the wage premium earned by students with an A relative to those who carry the lower grade B is eroded by grade inflation. While this effect reduces aggregate output and hurts able students, grade inflation allows low-ability students to grasp this "grade premium" with a certain probability, and so is beneficial for these students.

The second central theme of the paper is the impact of grade inflation on the job prospects of socially disadvantaged students. To analyse this issue, students are modelled to belong to two social classes, with the share of high-ability students being higher in the favoured class. This assumption reflects the stylised fact, well documented by many empirical studies (see, for example, Ermisch and Francesconi, 2001, and Blanden and Gregg, 2004), that students' performance and academic achievement are strongly influenced by family background. It is worth noting that it is not relevant for the present analysis whether this fact is due to a biased school system, insufficient resources, inadequate parenting, inherited genetic endowments, or any other cause (for an empirical analysis of economic vs. natural causes of scholarly achievement, see Plug and Vijverberg, 2003). Rather, the paper takes abilities as they occur at the time of the examination as given. Addressing the concern, mentioned above, that lower class students, on average, fare worse in tests at this age, the paper then asks whether such students benefit from reducing the information content of grades. This is not the case, however, as long as social origin is observable. Indeed, it is shown that the aggregate wage received by lower class workers is strictly decreasing in the extent of grade inflation. The reason is that, when grades do not fully reveal true abilities, employers use social origin as an additional source of information to update their beliefs about the expected productivity of a potential worker. With grade inflation, employers therefore discount the value of grade A earned by a lower-class student relative to the same grade earned by a higher-class student. Put differently, grade inflation induces firms to statistically discriminate against applicants from disfavoured origins.

The analysis shows that in a stable assignment, this effect results in a second wage premium, the "social premium", which is paid to A-students from the favourable social origin but not to lower-class A-students. In a further result, it is examined how the grade premium and the social premium together determine the preferences for grading policies in the student population. Specifically, it is shown that high-ability students from the favoured social origin, whom one might call the "elite", prefer grade inflation over honest grading if the reward for skill is concentrated in a few very productive jobs and if the skill composition of social classes differs strongly. In such circumstances, the social premium is particularly high so that securing it through grade inflation is worthwhile for the "elite". Intuitively, with inflated grades, able students from disadvantaged backgrounds cannot signal their ability anymore, and so can compete less successfully for the most attractive jobs, leaving more of those for the students from favoured backgrounds.

By addressing the link between degrees and the labour market, the paper contributes both to labour economics and to the economics of education. In labour economics, it has long been recognised that the assignment of workers to tasks affects wages (see the survey by Sattinger, 1993). For example, Teulings (1995) and Costrell and Loury (2004) provide general equilibrium models where, similar to the approach used here, high-ability workers have a comparative advantage in performing demanding jobs. As a main feature of assignment models, which the present paper shares, these contributions show that the wage differential obtained by high-ability workers does not only reflect productivity differences on any given job but depends on the matching between workers and tasks. Moreover, the assignment and the wages are affected by the information employers have about workers of different types. If such information is incomplete, by consequence, statistical discrimination affects the labour market outcome (see, for example, Coate and Loury, 1993, Norman, 2003, and Bjerk, 2008), just as in the present paper. To this literature, the present paper adds by highlighting the importance of school leaving grades and social origin for the matching and the wages obtained in equilibrium. It shows that different subgroups of the population are differently affected by the statistical discrimination introduced by grade inflation, thereby changing political preferences concerning the grading scheme to be employed. The paper so points out that the equilibrium assignment on the labour market may feed back into the schooling system.

An issue which raises concerns similar to those expressed about tough school leaving examinations are formal tests of job applicants administered by the employer. In an empirical study of the hiring decisions of a large retailing firm, Autor and Scarborough (2008) provide an analysis of the effects of such tests. They show that in this firm, as one might have expected, minority applicants fare worse than average in formal tests. However, introducing such tests did not decrease the chances of members of these groups to be hired. This result is due to an effect similar to the one analysed in the present paper: Like truthful grading, the formal test provides valuable information about the abilities of individuals and so reduces the scope for statistical discrimination against minority applicants. Thus, the result by Autor and Scarborough (2008) lends some empirical support for the theory laid out here. Its conceptual framework, however, is rather different, and arguably somewhat narrower, than the model presented here. In describing only the selection of workers for one type of job, these authors do not address the issue of assigning different workers to different tasks which is central to the present contribution. Moreover, contrary to Autor and Scarborough who focus on a single firm, the present approach endogenises wages in an equilibrium framework and so can explain wage differentials across grades and social origins.

In the theoretical research on education, grading policies so far have received rather little attention. In several classical contributions, Betts (1998a) and Costrell (1994, 1997) develop a model where grades are used to provide incentives for students so as to exert effort at school. This model has been extended by Himmler and Schwager (2007) who show that the social composition of a school determines its grading policy. In Jürges, Richter, and Schneider (2005), the role of testing is to provide incentives for teachers rather than for students. Mechtenberg (2009) analyzes the effect of teacher's bias in grading on career choices of students. She shows that such a bias can result in a gender specific allocation of students to fields which is not warranted by inherent differences in abilities. Finally, Chan, Hao, and Suen (2007) provide a signalling model where grade inflation affects the assignment of workers to tasks, and where grade inflation occurs in equilibrium as a result of cheap talk. To this model, the present paper adds a game theoretic foundation of the equilibrium assignment on the labour market. Moreover, both papers differ with regard to the information structure used. While Chan, Hao, and Suen (2007) focus on the asymmetric information between schools and firms regarding the grading scheme, in the present paper, the interaction of grades and social origin as signals of ability is at the centre of the analysis. Altogether, the specific contribution to education economics offered by the the present paper consists in clarifying the role of grades and social origin for the allocation on the labour market.

The paper is organised as follows. The model is presented in Section 2. In the following Section 3, the stable assignment on the labour market is characterised. Section 4 contains the results on the interaction of grade inflation and the wages received by the different subgroups of workers. Section 5 summarises and offers some lines for future research. Longer proofs are collected in the Appendix.

2 The Model

The model describes a labour market with a continuum of firms each of which offers one job and a continuum of workers. The sets of firms and workers are modelled as adjacent intervals on the real line, with $i \in [0, M)$ denoting firms, $j \in [M, M+N)$ denoting workers, and $k \in [0, M+N)$ being used for an agent of unspecified role. The lengths $M > 0$ and N, $0 < N < M$, of the intervals are the Lebesgue-measures of the sets of workers and firms.

Workers. Workers are characterised by their skill, or ability, $s \in \{a, b\}$, with a (b) denoting the higher (lower) skill level, and by their social origin, or class, $c \in \{h, \ell\}$, with h (ℓ) denoting that the worker comes from a socially favoured (disadvantaged) background. To refer to the ability and class of worker $j \in [M, M+N)$, I write $s(j)$ and $c(j)$. The numbers $n_{sc} > 0$, for $s = a, b$ and $c = h, \ell$, denote the measures of workers with skill s originating from class c. The measure of high- (low)- skilled workers is $n_a = n_{ah} + n_{a\ell}$ ($n_b = n_{bh} + n_{b\ell}$), and the measure of socially favoured (disadvantaged) workers is $n_h = n_{ah} + n_{bh}$ ($n_\ell = n_{a\ell} + n_{b\ell}$). The population measures satisfy $n_{ah} + n_{a\ell} + n_{bh} + n_{b\ell} = N$ and

Assumption 1 $n_{ah}/n_h > n_{a\ell}/n_\ell$.

Assumption 1 states that, while there are individuals of both skills in both social groups, the share of high-ability workers is larger in the socially favoured group than in the disadvantaged group. In view of Assumption 1, it is apparent that social background is not the only interpretation of the variable "class". More broadly, this variable can represent any characteristic which is not itself ability, but which, in the population as a whole, is correlated with ability or performance at the workplace.

As a student, before entering the workplace, each worker is given a grade $G \in \{A, B\}$, with $G(j)$ denoting the grade given to worker $j \in [M, M+N)$. The grade A (B) is meant to express that the student with this grade is of high (low) ability. Grading policy is captured by the fraction γ, $0 \leq \gamma \leq 1$, of students with ability b who nevertheless obtain grade A. Thus, a positive value of γ means that grades are inflated, while $\gamma = 0$ indicates "honest" grading. This modelling choice is simplified in two respects. Firstly, the converse of grade inflation would be that some truly excellent students are awarded only grade B. Such "grade deflation" does not seem to be a realistic case and is so ruled out for simplicity. Secondly, the extent of grade inflation is assumed to be the same throughout the student population, irrespective of social class. This assumption captures the idea that systematic discrimination along social origins inside the classroom will be detected by students, and so can be ruled out legally.

Combining grade and social origin, the set of workers is partitioned into four subsets. Depending on the grade inflation parameter γ, the measure of workers with grade G originating from class c is denoted by $N_{Gc}(\gamma)$, for $G = A, B$ and $c = h, \ell$. One has

$$N_{Ac}(\gamma) = n_{ac} + \gamma n_{bc} \quad \text{for } c = h, \ell \,, \tag{1}$$

$$N_{Bc}(\gamma) = (1-\gamma) n_{bc} \quad \text{for } c = h, \ell \,, \tag{2}$$

and the total measure of students with grade $G = A, B$ is denoted by $N_G = N_{Gh} + N_{G\ell}$.

Firms. The technology is described by two production functions $f_s : [0, M) \to \mathbb{R}_+$, for $s = a, b$, which are twice continuously differentiable. The value $f_s(i)$ gives the aggregate output produced by the firms in the interval $[0, i)$ if all of them employ a worker of skill s. Correspondingly, the derivative $f_s'(i)$ describes the output produced by firm i if its job is taken by a worker of ability s. The production functions satisfy $f_s(0) = 0$ for $s = a, b$

and

Assumption 2 (a) $f''_s(i) < 0$ *for* $s = a, b$ *and for all* $i \in [0, M)$,

(b) $f'_a(i) > f'_b(i) > 0$ *for all* $i \in [0, M)$,

(c) $f''_a(i) - f''_b(i) < 0$ *for all* $i \in [0, M)$.

Assumption 2 (a) states that firms are labelled in decreasing order of productivity. Such differences in productivity across firms can, for example, reflect different endowments of capital, different entrepreneurial abilities of the firms' owners, or different technologies. As expressed in Assumption 2 (b), workers of both skill levels are productive, but every firm produces more output with a high ability than with a low-ability worker. Finally, Assumption 2 (c) requires that the difference in output produced by workers of both skills decreases with the index of the firm. This means that skill differences matter more in highly productive than in less productive firms. This assumption formalises the idea that a firm with a sophisticated technology, high capital endowment, or a capable management can put employees' skills to more productive uses than a firm lacking such complementary inputs. In short, Assumption 2 requires that the marginal product of labour is decreasing (a), that high-ability workers have an absolute advantage in all firms (b), but that high- (low-) ability workers have a comparative advantage in high (low) productivity firms (c).

Firms cannot observe the true skill level s. They do observe, however, the grade G and the social origin c of the student as well as the aggregate extent of grade inflation γ. To motivate this informational structure, notice that it is fundamentally difficult for an outsider to assess the validity of an individual student's grade, whereas the aggregate grading policy may be known due to reputation effects. Moreover, it seems plausible that a personnel manager can quite easily find out about the social background of a job applicant, say by observing the name, address, manners, clothing, parents' occupation, etc. Based on (1) and (2) one finds the posterior probability $\text{Prob}\{s|G(j), c(j); \gamma\}$ that worker j is of skill $s = a, b$, conditional on her grade $G(j)$ and class $c(j)$. For the high skill level these probabilities are, for $c = h, \ell$, given by

$$\begin{aligned}
\text{Prob}\{a|A, c; \gamma\} &= \frac{n_{ac}}{N_{Ac}(\gamma)} = \frac{n_{ac}}{n_{ac} + \gamma n_{bc}} \stackrel{\text{def.}}{=} \rho_c(\gamma)\,, &(3)\\
\text{Prob}\{a|B, c; \gamma\} &= 0\,, &(4)
\end{aligned}$$

and for the low skill level they are $\text{Prob}\{b|A, c; \gamma\} = 1 - \rho_c(\gamma)$ and $\text{Prob}\{b|B, c; \gamma\} = 1$ for $c = h, \ell$. Notice that the grade B fully reveals ability b, so that social class does not matter for students with a B. Hence, there remain only the three signals $(A, h), (A, \ell), B$ which distinguish workers from the point of view of the firms.

The probabilities in (3) and (4) are used to evaluate the expected output

$$\varphi(i, j; \gamma) = \sum_{s=a,b} \text{Prob}\{s|G(j), c(j); \gamma\} \cdot f'_s(i) \tag{5}$$

worker j will produce in firm i if only her signal $(G(j), c(j))$ is known. For a student j with grade $G(j) = A$, from (3), this reduces to

$$\varphi(i, j; \gamma) = \rho_{c(j)} f'_a(i) + (1 - \rho_{c(j)}) f'_b(i)\,. \tag{6}$$

A student j with grade $G(j) = B$ is recognised as being of low ability and hence will be attributed the expected output

$$\varphi(i, j; \gamma) = f'_b(i)\,. \tag{7}$$

Matching. In order to describe the matching between workers and firms, it is convenient to re-arrange the names of workers after grading has occurred so that all workers with signal (A, h) are collected in the interval $J_{Ah} \stackrel{\text{def.}}{=} [M, M + N_{Ah})$, all workers with signal (A, ℓ) are located in $J_{A\ell} \stackrel{\text{def.}}{=} [M + N_{Ah}, M + N_A)$, and all workers with signal B are placed in the interval $J_B \stackrel{\text{def.}}{=} [M + N_A, M + N)$. Writing λ for the Lebesgue-measure defined for the Borel-sets contained in $[0, M + N)$, a feasible matching between firms and workers is then defined as follows.

Definition 1 *A feasible matching is a Lebesgue-measurable function* $\mu : [0, M + N) \to [0, M + N)$ *such that*

(a) $\mu(\mu(k)) = k$ *for all* $k \in [0, M+N)$,

(b) $\mu(i) \in [M, M+N) \cup \{i\}$ *for all* $i \in [0, M)$,

$\mu(j) \in [0, M) \cup \{j\}$ *for all* $j \in [M, M+N)$,

(c) *for all Lebesgue-measurable sets* $K \subset [0, M+N)$*, it holds* $\lambda(\mu(K)) = \lambda(K)$.

Definition 1 (a) imposes bilateral consistency on the matching function, stating that one's partner's partner is oneself. Part (b) of Definition 1 formalises the two-sidedness of the matching between firms and workers. If a firm $i \in [0, M)$ has a partner, then this must be a worker $j \in [M, M+N)$, and vice versa. If an agent k is unmatched I write $\mu(k) = k$. Finally, Definition 1 (c) says that for any set K of agents, the set of their partners must be of equal measure. This expresses, in the context of an atomless agent space, the idea that each firm offers just one job and each worker works in (at most) one firm. Requirement (c) is thus a market clearing condition stating that the matches formed must be feasible in the aggregate.

Stable assignment. Payoffs π_i and ω_j of firms and workers are generated from the matching function μ together with a wage function $w : [0, M) \to \mathbb{R}_+$ describing for each firm $i \in [0, M)$ the wage $w(i)$ it pays. If a firm has a partner, i.e., if $\mu(i) \neq i$ for some $i \in [0, M)$, its payoff is the expected output according to (5) net of the wage paid,

$$\pi_i = \varphi(i, \mu(i); \gamma) - w(i)\,. \tag{8}$$

If a worker has a job, i.e., if $\mu(j) \neq j$ for some $j \in [M, M+N)$, then her payoff is the wage paid by her employer,

$$\omega_j = w(\mu(j))\,. \tag{9}$$

The payoffs of unmatched agents are set to zero, that is for all $i \in [0, M)$ or $j \in [M, M+N)$ with $\mu(i) = i$ or $\mu(j) = j$, one has $\pi_i = 0$ and $\omega_j = 0$.

Definition 2 *A stable assignment is a feasible matching* μ*, a wage function* $w : [0, M) \to \mathbb{R}_+$ *and payoffs* π_i *for all* $i \in [0, M)$ *and* ω_j *for all* $j \in [M, M+N)$ *generated by* μ *and* w

such that

(a) $\pi_i \geq 0$ *for all* $i \in [0, M)$ *and* $\omega_j \geq 0$ *for all* $j \in [M, M+N)$

(b) $\pi_i + \omega_j \geq \varphi(i, j; \gamma)$ *for all* $i \in [0, M)$ *and* $j \in [M, M+N)$.

Part (a) of Definition 2 is an individual rationality constraint stating that no agent does worse in the matching than she would do by staying alone. Definition 2 (b) requires that the sum of the payoffs of any firm and any worker, whether matched together or not, is at least as large as the output this pair could produce together. This condition says that no pair has the potential to improve on their payoffs by matching together. An assignment is thus stable if there is no profitable unilateral or bilateral deviation from the allocation it induces.

In the following section, the set of stable assignments in the labour market is characterised.

3 The Labour Market

In two propositions, it is described how the information conveyed by grade and social origin determines the allocation on the labour market. The first of these propositions deals with the matching between workers and firms, and the second one gives the wages paid to different workers in a stable assignment. In both propositions, results are formulated for almost all workers or almost all firms, i.e., for all workers or firms except possibly a set of measure zero. This is because feasibility as defined in Definition 1 (c) only stipulates that measures of the sets of matched agents are equal, leaving the treatment of zero measure sets, and hence of individual agents, undetermined.

The first result states that the labour market is cleared:

Proposition 1 (Labour market clearing). *Consider* $\gamma > 0$ *and let a stable assignment* (μ, w) *be given. Then, for almost all workers* j *and almost all firms* i *one has:*

(a) *Workers from a favourable background with grade A are matched to firms $i \in [0, N_{Ah})$,*

$$\lambda(\{i \in [0, N_{Ah}) \,|\, \mu(i) \in J_{Ah}\}) = \lambda(\{j \in J_{Ah} \,|\, \mu(j) \in [0, N_{Ah})\}) = N_{Ah} \,.$$

(b) *Workers from an unfavourable background with grade A are matched to firms $i \in [N_{Ah}, N_A)$,*

$$\lambda(\{i \in [N_{Ah}, N_A) \,|\, \mu(i) \in J_{A\ell}\}) = \lambda(\{j \in J_{A\ell} \,|\, \mu(j) \in [N_{Ah}, N_A)\}) = N_{A\ell} \,.$$

(c) *Workers with grade B are matched to firms $i \in [N_A, N)$,*

$$\lambda(\{i \in [N_A, N) \,|\, \mu(i) \in J_B\}) = \lambda(\{j \in J_B \,|\, \mu(j) \in [N_A, N)\}) = N_B \,.$$

(d) *Firms $i \in [N, M)$ are unmatched,* $\lambda(\{i \in [N, M) \,|\, \mu(i) = i\}) = M - N \,.$

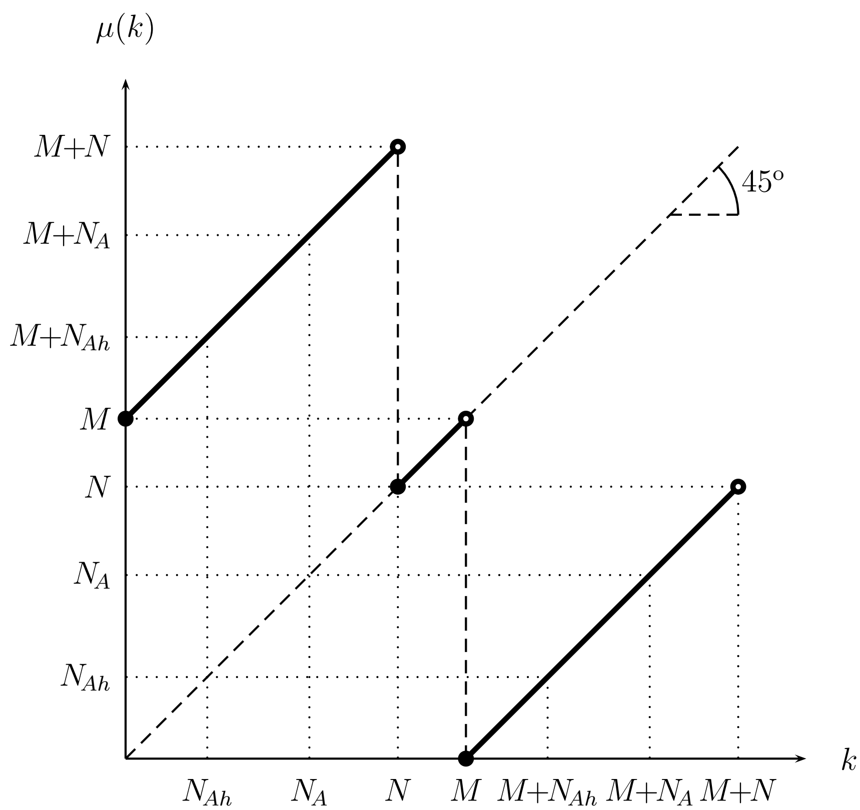
Proof. See Appendix. ■

According to Proposition 1, in a stable assignment the allocation of workers to firms is governed by the three signals (A, h), (A, ℓ), and B, which partition students in a three-layer hierarchy. The A-students from the higher social class work in the N_{Ah} most productive jobs; the A-students from the lower class are employed in the next $N_{A\ell}$ somewhat less productive firms; and the remaining students, who have grade B, are active in the following N_B even less productive firms. Finally, since the measure of workers N is less than the measure of firms M, market clearing entails that there remains a set of measure $M - N$ of vacant jobs, which are in the least productive firms.

This matching pattern is illustrated in Figure 1. In this figure, both axes display the set of agents, starting with firms in the interval $[0, M)$ and continuing with workers in the interval $[M, M + N)$. Every point in this diagram corresponds to a potential match. The bold line is the graph of the function

$$\mu(k) = \begin{cases} M + k & \text{if} \quad 0 \leq k < N \\ k & \text{if} \quad N \leq k < M \\ k - M & \text{if} \quad M \leq k < M + N \,, \end{cases}$$

Figure 1: **Matching in a stable assignment.**



which provides an example for a feasible matching function satisfying Definition 1 and displaying the properties described in Proposition 1. Starting at $k = 0$, the function increaseses as long as $k < N$, illustrating the fact that as productivity declines, firms obtain workers with lower signals of ability. For $k \in [N, M)$, the graph is on the diagonal, since the least productive jobs are unfilled. For $k \in [M, M+N)$, from bilateral consistency, the graph is the image of the first part mirrored at the diagonal. This again illustrates, now from the workers' point of view, the monotonic relationship between signal and job quality.

The following proposition quantifies the signal-specific wages obtained in a stable assignment.

Proposition 2 (Wages). *Consider $\gamma > 0$ and let a stable assignment (μ, w) be given. Then, almost all workers $j \in J_S$, for $S = Ah, A\ell, B$, obtain a wage $w(\mu(j)) = w_S(\gamma)$*

given by

$$
\begin{aligned}
\text{(a)} \quad w_B(\gamma) &= f_b'(N) \overset{def.}{=} w_B \\
\text{(b)} \quad w_{A\ell}(\gamma) &= w_B + \rho_\ell(\gamma)[f_a'(N_A) - f_b'(N_A)] \\
\text{(c)} \quad w_{Ah}(\gamma) &= w_{A\ell}(\gamma) + [\rho_h(\gamma) - \rho_\ell(\gamma)][f_a'(N_{Ah}) - f_b'(N_{Ah})]
\end{aligned}
$$

Proof. See Appendix. ∎

As Proposition 2 shows, in a stable assignment, wages are completely determined by the signals which workers convey about their productivity. Specifically, as can be seen by comparing expressions (a) and (b) in this proposition, the students with signal (A, ℓ) obtain a wage premium compared to the students with the bad grade B. This premium expresses the market valuation of the better grade and is therefore called the

$$\text{grade premium:} \quad \text{GRA } (\gamma) \overset{\text{def.}}{=} \rho_\ell(\gamma)[f_a'(N_A(\gamma)) - f_b'(N_A(\gamma))] \,. \tag{10}$$

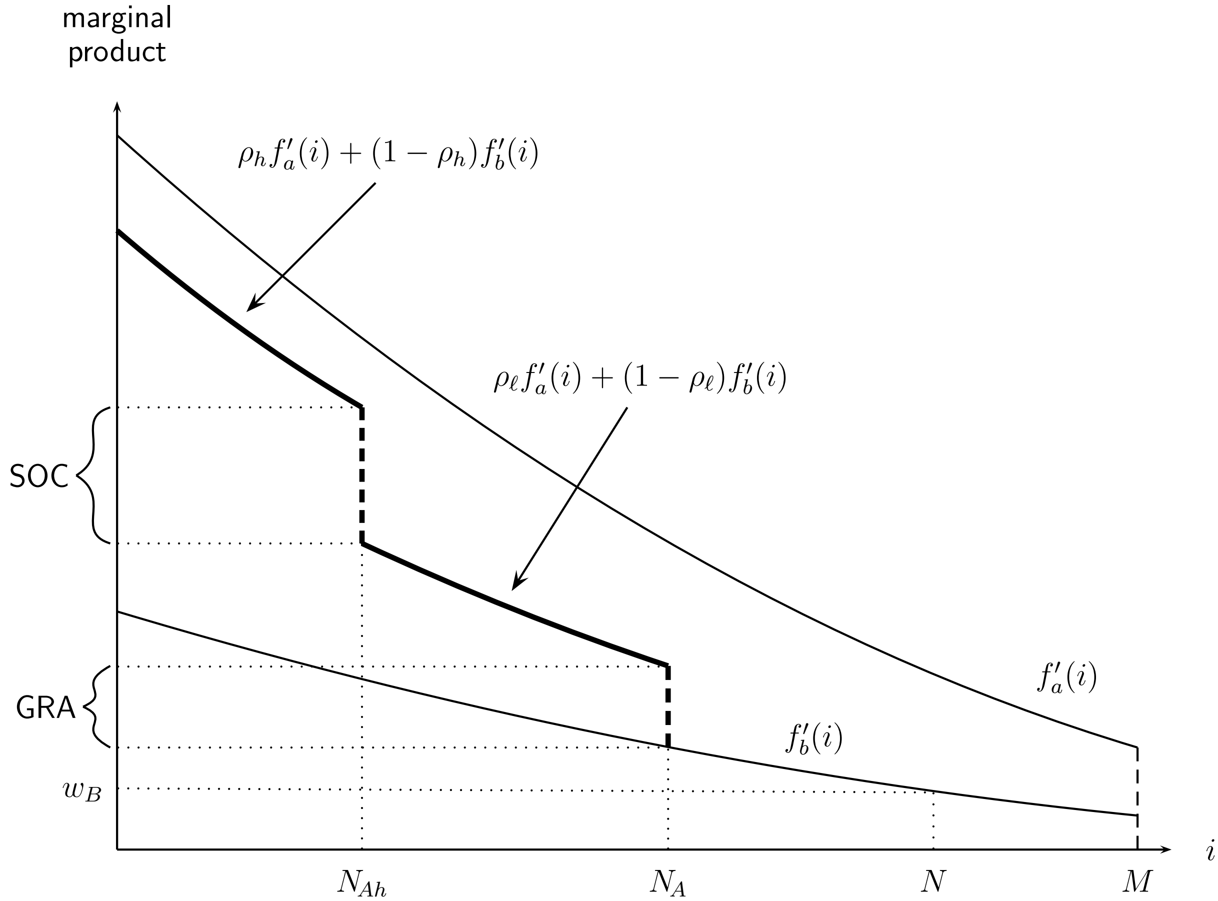
Its value is determined by the productivity differential of the last A-student times the probability that this student is in fact of high ability. Since high-ability students are more productive than low-ability students (Assumption 2 b), the grade premium is positive for all $0 < \gamma \leq 1$.

Similarly, from Proposition 2 (b) and (c), the A-students originating from the higher social class obtain a wage premium relative to the lower class A-students, called the

$$\text{social premium:} \quad \text{SOC } (\gamma) \overset{\text{def.}}{=} [\rho_h(\gamma) - \rho_\ell(\gamma)][f_a'(N_{Ah}(\gamma)) - f_b'(N_{Ah}(\gamma))] \,. \tag{11}$$

Assumption 1 implies that $\rho_h(\gamma) > \rho_\ell(\gamma)$ if $\gamma > 0$. With Assumption 2 (b) it then follows that the social premium is strictly positive if $\gamma > 0$. This shows that, in a situation where grades are inflated, employers interpret a favourable social origin as an additional signal for high ability. In the market, this valuation by firms is cashed in by the higher class A-students in the form of a wage differential relative to their lower class counterparts. Its value is the product of the productivity differential between both skill levels, evaluated at

Figure 2: **Wages in a stable assignment.**



the last (A, h)-student, times the difference in posteriors attached to A-students of both classes. It so expresses by how much the expected productivity of a higher-class A-student exceeds the one of a lower-class A-student.

The premia are illustrated in Figure 2. There, the output produced by high skilled and low skilled workers is plotted against the index of firms $i \in [0, M)$ in form the two marginal product curves f_a' and f_b'. Averaging these marginal products with the class-specific posterior ρ_h (ρ_ℓ), one obtains the upper (lower) bold line which gives the expected output of a higher (lower) class worker with grade A. The grade premium is such that the last firm able to hire an A-student, i.e., firm $i = N_A$, is indifferent between hiring this student and a student of low ability. Geometrically, the grade premium is then the difference between the lower bold curve and the marginal product f_b', both evaluated at N_A. Similarly, the social premium is determined by the choice of the last firm able to find an upper class student with an A, i.e., firm $i = N_{Ah}$. This firm must be indifferent between hiring such a student and an A-student from the disfavoured origin. The difference between the expected productivities of these two students, geometrically expressed by the vertical jump between both bold curves occurring at N_{Ah}, is the social premium.

To conclude this section, a remark is in order concerning the behaviour of the model when $\gamma = 0$. With honest grading, the three signals collapse to two which are exclusively defined by the grades A and B. This occurs because in this case, the grade A fully reveals high ability so that there is no reason to differentiate workers with this grade according to social origin. Because of this discontinuity, Propositions 1 to 2 cannot literally be extended to $\gamma = 0$. However, one can easily state modified versions of these propositions for this case, with the wage for the high grade being $w_A = w_B + f'_a(n_a) - f'_b(n_a)$. Now the grade premium converges to $f'_a(n_a) - f'_b(n_a)$ and the social premium converges to zero if $\gamma \to 0$. Hence, the wages are continuous at $\gamma = 0$, even though the underlying matching need not be.

In the following section, it is analysed how the wages are affected by changes in the grade inflation parameter, and which students benefit from such changes.

4 Winners and Losers From Grade Inflation

Using Proposition 2, one can derive the expected wages w_{sc}, $s = a, b$ and $c = \ell, h$, obtained by the four different types of workers in the economy. Since students with a high ability certainly obtain grade A, the wage of high-ability students originating from social class $c = h, \ell$ is the wage paid to A-students from this class, $w_{ac}(\gamma) = w_{Ac}(\gamma)$. For low-ability students the expected wage is a weighted average of the wage obtained by B- and by A-students of the corresponding origin, $w_{bc}(\gamma) = (1 - \gamma)w_B + \gamma w_{Ac}(\gamma)$ for $c = \ell, h$.

In order to analyse which group, if any, benefits from a deviation from honest grading, I consider for each type (s, c), for $s = a, b$ and $c = h, \ell$, the grade inflation parameter $\gamma_{sc} \stackrel{\text{def.}}{=} \text{argmax}_{\gamma}\{w_{sc}(\gamma)|0 \leq \gamma \leq 1\}$ which is most preferred by this type. In order to characterise the preferred grading policies, the following derivatives, which can straightforwardly be calculated from Proposition 2, are useful.

$$
\begin{aligned}
w'_{a\ell}(\gamma) &= \text{GRA}'(\gamma) && (12)\\
w'_{b\ell}(\gamma) &= \gamma\text{GRA}'(\gamma) + \text{GRA}(\gamma) && (13)\\
w'_{ah}(\gamma) &= \text{GRA}'(\gamma) + \text{SOC}'(\gamma) && (14)\\
w'_{bh}(\gamma) &= \gamma[\text{GRA}'(\gamma) + \text{SOC}'(\gamma)] + [\text{GRA}(\gamma) + \text{SOC}(\gamma)] && (15)
\end{aligned}
$$

From (10) and (11), the derivatives of the premia featuring in (12) to (15) are

$$
\begin{aligned}
GRA'(\gamma) &= \rho'_{\ell}(\gamma)\Big[f'_a(N_A(\gamma)) - f'_b(N_A(\gamma))\Big] && (16)\\
&\quad + \rho_{\ell}(\gamma)\Big[f''_a(N_A(\gamma)) - f''_b(N_A(\gamma))\Big]\frac{\partial N_A}{\partial \gamma}\\
SOC'(\gamma) &= \Big[\rho'_h(\gamma) - \rho'_{\ell}(\gamma)\Big]\Big[f'_a(N_{Ah}(\gamma)) - f'_b(N_{Ah}(\gamma))\Big] && (17)\\
&\quad + \Big[\rho_h(\gamma) - \rho_{\ell}(\gamma)\Big]\Big[f''_a(N_{Ah}(\gamma)) - f''_b(N_{Ah}(\gamma))\Big]\frac{\partial N_{Ah}}{\partial \gamma}\,.
\end{aligned}
$$

Consider first the low-ability students of both social origins. When determining her preferred grading policy, such a student faces a trade-off between two effects of an increase in grade inflation expressed by the two terms on the right-hand-sides of (13) and (15). On the one hand, with increased grade inflation, it becomes more likely that this student obtains an undeserved A, and so it becomes more likely that she receives the grade premium and, if she is of the higher class, the social premium (the second terms in (13) and (15)). Since the premia are positive, from this effect low-ability students would like to see more grade inflation. On the other hand, the premia themselves are affected if more workers with grade A enter the labour market (the first terms in (13) and (15)). Low-ability students care about this effect since, in a situation with grade inflation, they sometimes also obtain an A. Therefore, also low-ability students do not necessarily want to push grade inflation ever further. If in the initial situation grading is honest, however, the second effect is not relevant since a low-ability student does not get any premium in the first place. By consequence, at $\gamma = 0$ the positive effect dominates so that a little grade inflation is always beneficial for low skilled students (see Proposition 3 a).

Turning now to the high-ability students from the lower social class, one sees from (12) that their preference is determined by the change in the grade premium $\mathrm{GRA}'(\gamma)$ induced by stronger grade inflation. As is evident from (16), this change occurs through two channels. Firstly, when employers assess an A-student, they attach a lower posterior to her being of truly high skill, $\rho'_\ell(\gamma) < 0$, since there are more low-ability students in the pool of those who are awarded an A. Secondly, from Assumption 2 (c), also the productivity differential $f'_a(N_A) - f'_b(N_A)$ decreases since the total number N_A of A-students increases. Thus, through both channels, the grade premium is eroded if grades are inflated, $\mathrm{GRA}'(\gamma) < 0$ for all $0 \leq \gamma \leq 1$. This implies that any increase in grade inflation makes the able, lower class students definitely worse off, as is stated in Proposition 3 (b).

It remains to assess the preference of the high skilled students from favourable backgrounds. Part (c) of Proposition 3 provides a sufficient condition implying that these students prefer some grade inflation over honest grading.

Proposition 3 (Preferences for grade inflation).

(a) *Low-ability students prefer grade inflation,* $\gamma_{bc} > 0$ *for both* $c = h, \ell$.

(b) *High-ability students from a disfavoured background prefer honest grading,* $\gamma_{a\ell} = 0$.

(c) *High-ability students from a favoured background prefer grade inflation,* $\gamma_{ah} > 0$, *if*

$$\begin{aligned} &\frac{n_{b\ell}}{n_{a\ell}}\Big\{[f'_a(n_{ah}) - f'_b(n_{ah})] - [f'_a(n_a) - f'_b(n_a)] + n_{a\ell}[f''_a(n_a) - f''_b(n_a)]\Big\} \\ &\quad > \frac{n_{bh}}{n_{ah}}\Big\{[f'_a(n_{ah}) - f'_b(n_{ah})] - n_{ah}[f''_a(n_a) - f''_b(n_a)]\Big\}. \end{aligned} \tag{18}$$

Proof. Parts (a) and (b) follow from the arguments in the text. To prove part (c), compute $w'_{ah}(0)$ according to (14), (16), and (17). At $\gamma = 0$, one has $\rho'_c(0) = -n_{bc}/n_{ac}$

and $\rho_c(0) = 1$ for $c = \ell, h$. Using this together with (1), one arrives at

$$\begin{aligned} w'_{ah}(0) \;=\;& \frac{n_{b\ell}}{n_{a\ell}}\Big\{[f'_a(n_{ah}) - f'_b(n_{ah})] - [f'_a(n_a) - f'_b(n_a)]\Big\} \\ & + [f''_a(n_a) - f''_b(n_a)](n_{bh} + n_{b\ell}) \;-\; \frac{n_{bh}}{n_{ah}}[f'_a(n_{ah}) - f'_b(n_{ah})]\,. \end{aligned}$$

Rearranging, one sees that $w'_{ah}(0) > 0$ is equivalent to (18). ■

While the first two results in Proposition 3 correspond to intuition and probably to the experience of most teachers, it is rather surprising that high-ability students of favourable origin, whom one may call the "elite", opt for grade inflation if condition (18) is satisfied (part c). To understand this condition, observe that its two sides describe how, starting from honest grading ($\gamma = 0$), a small amount of grade inflation changes the wage of ah-students by affecting competition with the other students. On the r.h.s., one has the erosion of the premia induced by the fact that grade inflation gives an undeserved A to some bh-students. This reduces the social premium by the productivity difference $f'_a(n_{ah}) - f'_b(n_{ah})$ times the change in the posterior $\rho'_h(0) = -n_{bh}/n_{ah}$. Moreover, since there are more A-students around, the grade premium is reduced by the change in productivity difference $f''_a(n_a) - f''_b(n_a)$ times the marginal change $\partial N_A/\partial\gamma = n_{bh}$ in the measure of A-students from the higher class. Thus, from the competition of bh-students alone, grade inflation is unequivocally costly to the "elite".

The l.h.s. of (18) gives the impact of grade inflation on the wage of ah-students transmitted by competition with the lower class students. Here, the negative term $-(n_{b\ell}/n_{a\ell})[f'_a(n_a) - f'_b(n_a)] + n_{b\ell}[f''_a(n_a) - f''_b(n_a)]$ again describes the erosion of the grade premium, which consists of two parts since the posterior ρ_ℓ and the productivity difference $f'_a - f'_b$ are both affected if some $b\ell$-students obtain an A. However, there is also a positive effect of grade inflation on "elite" wages which is captured by $(n_{b\ell}/n_{a\ell})[f'_a(n_{ah}) - f'_b(n_{ah})]$. This term expresses the amount of social premium gained by introducing grade inflation. It is composed of $n_{b\ell}/n_{a\ell}$, the decrease in posterior ρ_ℓ after γ is raised above zero, times the productivity difference $f'_a(n_{ah}) - f'_b(n_{ah})$. If this effect is strong enough, grade inflation may benefit the high-ability students from the upper social class.

To illustrate the circumstances leading to this result it is instructive to see which production functions and values of the fundamental parameters satisfy (18). Observe that $f'_a(n_a) - f'_b(n_a) - n_{a\ell}[f''_a(n_a) - f''_b(n_a)]$ is a linear approximation of $f'_a(n_{ah}) - f'_b(n_{ah})$, so that the l.h.s. of (18) is strictly positive if the productivity difference $f'_a - f'_b$ is a strictly convex function. If in addition, n_{bh}/n_{ah} is small enough relative to $n_{b\ell}/n_{a\ell}$, then the inequality holds true. Thus, the high-ability students from a favoured background prefer grade inflation if both $f'_a - f'_b$ is strictly convex and the skill composition differs widely between the social classes.

Clearly, this latter property implies that employers put a high weight on class when evaluating the expected productivity of applicants, so that the social premium is particularly high. To see why also a convex productivity difference raises the social premium, notice that with honest grading, employers recognise that the $a\ell$-students are just as productive as the ah-students. Honest grading therefore raises the number of students who display the best signal available in the market and thereby depresses the wage obtainable by the ah-students who hold this signal. Put differently, grade inflation shields the highly skilled students with upper class backgrounds from competition by the equally able students originating from disadvantaged social classes. Now a convex function $f'_a - f'_b$ means that the productivity difference between both ability levels first decreases sharply in the number of high skilled workers employed, and slowly later on. Thus, the reward for high skill is concentrated among a few very highly productive jobs, for example in finance or in top management, while it erodes fast if more high skilled workers enter the labour market. Therefore, when the productivity difference between skills is convex, restricting competition by the lower classes with the help of grade inflation is particularly rewarding for the "elite" students, who can in this way ensure that the jobs at the very top remain reserved for them.

In addition to type-specific preferences for grade inflation as analysed in Proposition 3, one might as well be interested in aggregate welfare measures. First, as a reference, expected aggregate output

$$F(\gamma) \stackrel{\text{def.}}{=} \rho_h f_a(N_{Ah}) + (1-\rho_h) f_b(N_{Ah}) \\ + \rho_\ell [f_a(N_A) - f_a(N_{Ah})] + (1-\rho_\ell)[f_b(N_A) - f_b(N_{Ah})] + f_b(N) - f_b(N_A) \tag{19}$$

produced in a stable assignment is considered. This corresponds to a utilitarian welfare function where the wages of all workers and the profits of all firms enter with equal weights. The following proposition notes that grade inflation is unambiguously detrimental to welfare defined in this way.

Proposition 4 (Efficiency loss from grade inflation).
Expected aggregate output is higher under honest grading than under grade inflation, $F(0) > F(\gamma)$ *for all* $\gamma > 0$.

Proof. See Appendix. ■

This result arises from the mismatch on the labour market induced by giving the high grade to some mediocre students. Since firms cannot distinguish between deserved and undeserved A's, the resulting allocation does not respect comparative advantage anymore and so is inefficient.

In social policy one is, however, quite often willing to accept efficiency losses so as to promote equity goals. In the present context, one might be ready to forgo some output if lenient grading were to raise the wages of the students from disfavoured backgrounds. To see whether this is the case, define

$$W_c(\gamma) \stackrel{\text{def.}}{=} n_{ac} w_{ac}(\gamma) + n_{bc} w_{bc}(\gamma) \tag{20}$$

as the aggregate wage income received by workers from social origin $c = h, \ell$.

Proposition 5 (Grade inflation and social origin).
The aggregate wage of lower class workers strictly decreases if grades are inflated more

strongly:

$$W_\ell'(\gamma) < 0 \quad \textit{for all } 0 \leq \gamma \leq 1\,.$$

Proof. Replacing $w_{a\ell}(\gamma) = w_{A\ell}(\gamma)$ and $w_{b\ell}(\gamma) = \gamma w_{A\ell} + (1-\gamma)w_B$ with the help of Proposition 2 and using (3), equation (20) becomes $W_\ell(\gamma) = n_{a\ell}[f_a'(N_A) - f_b'(N_A)] + n_\ell w_B$. Noting $\partial N_A/\partial\gamma = n_b$, differentiating yields

$$\frac{\partial W_\ell(\gamma)}{\partial\gamma} = n_{a\ell}n_b[f_a''(N_A) - f_b''(N_A)] < 0\,,$$

with the sign following on Assumption 2 (c). ∎

Proposition 5 says that grade inflation unambiguously hurts the lower classes, and that, consequently, honest grading is not only optimal if one wants to maximise output, but also if one wants to promote the welfare of socially disadvantaged students. At the heart of this result is the informational structure of the present model. Lower class students are less able on average, so that one might think that camouflaging academic performance by inflated grades allows these students to grasp some jobs which otherwise would be reserved to students with proven skills. This argument, however, misses the fact that social origin is observable irrespective of the grading scheme. Thus, employers cannot be fooled into disregarding the lower average ability of disadvantaged students. The only result achieved by grade inflation is that the assessment of abilities is distributed differently among the lower class students. The low-ability students from this class gain by grade inflation, but this gain comes exclusively at the expense of the high-ability students of the same background. In addition, the efficiency loss induced by grade inflation implies that wage income is not only redistributed among the lower class agents but decreases overall.

5 Conclusion

In this paper, it was analysed how a grading policy which awards good grades to mediocre students affects the labour market. It was shown that such grade inflation benefits students of low ability, but that it reduces output and hurts students from disadvantaged social backgrounds. Moreover, grade inflation may benefit students from favoured social origins because it devalues the good grades earned by aspiring lower class students, thereby reducing competition for attractive jobs. For education policy, these results suggest that lenient grading is not a suitable means for promoting equality of opportunities. Quite the contrary, a strictly meritocratic system where performance is measured and documented objectively is the best way to induce social mobility.

These results were derived in a model of two-sided matching between firms and workers. This approach, which provides a microeconomic foundation for the allocative role of grades, lends itself to a number of extensions, two of which I briefly discuss. Firstly, it seems natural to extend the model of a centralised grading authority presented in the present paper to a model where several schools, or school districts, set their own grading policies. In such a model one could answer questions such as whether decentralised grades are necessarily, as is often claimed, less stringent than centrally chosen grades, or whether decentralisation hurts low-ability and/or lower class students. As a second extension, one might consider to integrate the matching model with a signalling approach where the grading policy of the schooling system is not observable. It would be an interesting, though challenging, task to find out whether honest grading can survive in an equilibrium in such a model.

Appendix

Proof of Proposition 1. I first prove that almost all firms $i < N$ are matched to workers,

$$\lambda(\{i \in [0, N) \,|\, \mu(i) \in [M, M+N)\}) = N\,. \tag{A.1}$$

Assume that (A.1) is false. Then there is a set of firms $I \subset [0, N)$ with positive measure $\lambda(I) > 0$ such that $\mu(I) \cap [M, M+N) = \emptyset$. Notice that this implies $\mu(i) = i$ and hence $\pi_i = 0$ for all $i \in I$. From the feasibility requirement in Definition 1 (c), there must be a set of workers $J \subset [M, M+N)$ of equal measure $\lambda(J) = \lambda(I)$ who are not matched to firms in $[0, N)$, i.e., $\mu(J) \cap [0, N) = \emptyset$, and hence $\mu(J) \subset J \cup [N, M)$. Thus, there must be a worker $j \in J$ who has no job, $\mu(j) = j$, or who is matched to a firm $\mu(j) \in [N, M)$.

Consider first the possibility $\mu(j) = j$. This implies $\omega_j = 0$. Since for any $i \in I$, one has $\pi_i = 0$, the stability requirement in Definition 2 (b) implies $0 = \pi_i + \omega_j \geq \varphi(i, j; \gamma)$. Since expected output must be strictly positive from Assumption 2 (b), one arrives at a contradiction.

Consider now the possibility $\mu(j) \in [N, M)$. Individual rationality in Definition 2 (a) implies $\pi_{\mu(j)} = \varphi(\mu(j), j; \gamma) - w(\mu(j)) \geq 0$. For any $i \in I$, one must then have $\pi_i + \omega_j = 0 + w(\mu(j)) \leq \varphi(\mu(j), j; \gamma)$. Moreover, since $i < N \leq \mu(j)$, it follows from Assumption 2 (a) that $\varphi(\mu(j), j; \gamma) < \varphi(i, j; \gamma)$. Hence, one arrives at $\pi_i + \omega_j < \varphi(i, j; \gamma)$, contradicting the stability requirement in Definition 2 (b). This establishes (A.1).

(a) The first equality follows from noting that by bilateral consistency in Definition 1 (a), $\{j \in J_{Ah} \,|\, \mu(j) \in [0, N_{Ah})\} = \mu(\{i \in [0, N_{Ah}) \,|\, \mu(i) \in J_{Ah}\})$ and hence, from feasibility in Definition 1 (c), both sets must have equal measure. To prove the second equality, assume to the contrary that this measure is strictly less than N_{Ah}. Then from (A.1), there is a set $I \subset [0, N_{Ah})$ with positive measure $\lambda(I) > 0$ such that $\mu(I) \subset J_{A\ell} \cup J_B$. Thus, there must be a set $J \subset J_{Ah}$ of workers with the same measure $\lambda(J) = \lambda(I)$ who cannot find a partner among the firms $i \in [0, N_{Ah})$. From (A.1), almost all of these workers are matched to some firms, and hence one can choose $j \in J \subset J_{Ah}$ such that $\mu(j) \in [N_{Ah}, N)$. Choose in addition $i \in I$ with the partner $\mu(i) \in J_{A\ell} \cup J_B$.

From Definition 2 (b), stability requires $\pi_i + \omega_j \geq \varphi(i, j; \gamma)$ and $\pi_{\mu(j)} + \omega_{\mu(i)} \geq \varphi(\mu(j), \mu(i); \gamma)$. Replacing the payoffs by (8) and (9) shows that these two inequalities are equivalent to $\varphi(i, \mu(i); \gamma) - w(i) + w(\mu(j)) \geq \varphi(i, j; \gamma)$ and $\varphi(\mu(j), j; \gamma) - w(\mu(j)) + w(i) \geq \varphi(\mu(j), \mu(i); \gamma)$. Adding these inequalities, one arrives at

$$\varphi(i, \mu(i); \gamma) + \varphi(\mu(j), j; \gamma) \geq \varphi(i, j; \gamma) + \varphi(\mu(j), \mu(i); \gamma) \,. \tag{A.2}$$

One has either $\mu(i) \in J_{A\ell}$ or $\mu(i) \in J_B$.

Consider first $\mu(i) \in J_B$. Using (6) and (7), one finds that (A.2) is equivalent to $\rho_h[f'_a(\mu(j)) - f'_b(\mu(j))] \geq \rho_h[f'_a(i) - f'_b(i)]$. From $\rho_h > 0$ and Assumption 2 (c), this implies $\mu(j) \leq i$. However, since $i \in I \subset [0, N_{Ah})$ and $\mu(j) \in [N_{Ah}, N)$, one also has $i < N_{Ah} \leq \mu(j)$, a contradiction.

Consider now $\mu(i) \in J_{A\ell}$. Using (6), one derives after some manipulations, which are available form the author upon request, that (A.2) is equivalent to

$$\gamma(n_{ah}n_{b\ell} - n_{bh}n_{a\ell})[f'_a(\mu(j)) - f'_b(\mu(j))] \geq \gamma(n_{ah}n_{b\ell} - n_{bh}n_{a\ell})[f'_a(i) - f'_b(i)]\,. \tag{A.3}$$

From Assumption 1, one has $n_{ah}n_{b\ell} > n_{bh}n_{a\ell}$. With $\gamma > 0$ it follows that (A.3) is equivalent to $f'_a(\mu(j)) - f'_b(\mu(j)) \geq f'_a(i) - f'_b(i)$. From Assumption 2 (c), this again implies $\mu(j) \leq i$, leading to the same contradiction.

(b) and **(c)** The proofs of claims (b) and (c) parallel the one of claim (a) and so are omitted for brevity.

(d) From (A.1) and feasibility in Definition 1 (c), there is at most a zero measure set of workers left for the firms $i \in [N, M)$, $\lambda(\{i \in [N, M) \,|\, \mu(i) \in [M, M+N)\}) = 0$. The complement set of unmatched firms must therefore have full measure, $\lambda(\{i \in [N, M) \,|\, \mu(i) = i\}) = M - N$. ■

Proof of Proposition 2. From Proposition 1, almost all workers are employed. Consider two such workers $j, j' \in J_S$. From the definition of payoffs in (8) and (9) and the stability requirement in Definition 2 (b), one has

$$\pi_{\mu(j)} + \omega_{j'} = \varphi(\mu(j), j; \gamma) - w(\mu(j)) + w(\mu(j')) \geq \varphi(\mu(j), j'; \gamma)\,. \tag{A.4}$$

Since the workers share the same signal, expected output would be the same if they worked both in the same firm $\mu(j)$, i.e., $\varphi(\mu(j), j'; \gamma) = \varphi(\mu(j), j; \gamma)$. Hence, the inequality in (A.4) implies $w(\mu(j')) \geq w(\mu(j))$. From the stability requirement $\pi_{\mu(j')} + \omega_j \geq \varphi(\mu(j'), j; \gamma)$ one derives in the same way that $w(\mu(j)) \geq w(\mu(j'))$. Altogether, one must have $w(\mu(j)) = w(\mu(j'))$, so that there is a signal-specific wage $w_S(\gamma)$ such that $w(\mu(j)) = w_S(\gamma)$ for almost all $j \in J_S$.

(a) To derive the wage $w_B(\gamma)$, observe that from Proposition 1 (c), there exists a sequence of firms $i \in [N_A, N)$ converging to N such that all i in the sequence are matched to workers $\mu(i) \in J_B$. From the argument above, for all those i, one has $w(i) = w_B(\gamma)$. Individual rationality in Definition 2 (a) requires for all those i that $\pi_i = \varphi(i, \mu(i); \gamma) - w(i) = f_b'(i) - w_B(\gamma) \geq 0$, where the second equality follows from (7). Taking the limit as $i \rightarrow N$ yields $w_B(\gamma) \leq f_b'(N)$.

From Proposition 1 (d), one can find a sequence of unmatched firms $i \in [N, M)$ converging to N. Note that for all i in this sequence, one has $\pi_i = 0$, and take some $j \in J_B$ who is matched to a firm so that $w(\mu(j)) = w_B(\gamma)$. Then, stability in Definition 2 (b) implies together with (7): $\pi_i + \omega_j = 0 + w_B(\gamma) \geq \varphi(i, j; \gamma) = f_b'(i)$. Taking the limit as $i \rightarrow N$, one finds $w_B(\gamma) \geq f_b'(N)$. Thus, $w_B(\gamma) = f_b'(N)$.

(b) From Proposition 1 (b), there exists a sequence of firms $i \in [N_{Ah}, N_A)$ converging to N_A such that all i in the sequence are matched to workers $\mu(i) \in J_{A\ell}$. From the argument above, all those firms pay the same wage $w_{A\ell}(\gamma)$ and so obtain payoff $\pi_i = \varphi(i, \mu(i); \gamma) - w_{A\ell}(\gamma)$. Consider a worker $j \in J_B$ who is matched to some firm and obtains payoff $\omega_j = w_B$. From stability in Definition 2 (b), one must have $\pi_i + \omega_j = \varphi(i, \mu(i); \gamma) - w_{A\ell}(\gamma) + w_B \geq \varphi(i, j; \gamma)$. From this inequality, one derives with the help of (6), (7), and $c(\mu(i)) = \ell$ that $w_{A\ell}(\gamma) \leq w_B + \rho_\ell(\gamma)[f_a'(i) - f_b'(i)]$. Taking the limit as $i \rightarrow N_A$, one arrives at $w_{A\ell}(\gamma) \leq w_B + \rho_\ell(\gamma)[f_a'(N_A) - f_b'(N_A)]$.

From Proposition 1 (c), one can find a sequence of firms $i \in [N_A, N)$ such that all i in the sequence are matched to workers $\mu(i) \in J_B$ and such that the sequence converges to N_A. For all those i, one has $\pi_i = \varphi(i, \mu(i); \gamma) - w_B$. Consider some worker $j \in J_{A\ell}$ who has a job and so earns $\omega_j = w(\mu(j)) = w_{A\ell}(\gamma)$. Stability requires $\pi_i + \omega_j = \varphi(i, \mu(i); \gamma) - w_B + w_{A\ell}(\gamma) \geq \varphi(i, j; \gamma)$. Inserting (7), (6), and $c(j) = \ell$, one finds $w_{A\ell}(\gamma) \geq w_B + \rho_\ell(\gamma)[f_a'(i) - f_b'(i)]$. In the limit for $i \rightarrow N_A$, one so has $w_{A\ell}(\gamma) \geq w_B + \rho_\ell(\gamma)[f_a'(N_A) - f_b'(N_A)]$ which establishes the claim.

(c) The proof is similar to the proof of claim (b) and is so omitted. ∎

Proof of Proposition 4. Using $\gamma = 0$ in (19), one finds that aggregate output in case of honest grading is $f_a(n_a) + f_b(N) - f_b(n_a)$. Comparison with (19) for $\gamma > 0$ shows that $F(0) > F(\gamma)$ if and only if

$$\begin{aligned} & f_a(n_a) - f_b(n_a) - \rho_h[f_a(N_{Ah}) - f_b(N_{Ah})] \\ > \;& \rho_\ell[f_a(N_A) - f_b(N_A)] - \rho_\ell[f_a(N_{Ah}) - f_b(N_{Ah})]\,. \end{aligned} \tag{A.5}$$

Now since $f_a - f_b$ is strictly concave from Assumption 2 (c), we have with $n_{ah} = \rho_h N_{Ah}$ and $f_a(0) - f_b(0) = 0$ that $\rho_h[f_a(N_{Ah}) - f_b(N_{Ah})] < f_a(n_{ah}) - f_b(n_{ah})$. Hence (A.5) is true if

$$f_a(n_a) - f_b(n_a) - [f_a(n_{ah}) - f_b(n_{ah})] > \rho_\ell[f_a(N_A) - f_b(N_A)] - \rho_\ell[f_a(N_{Ah}) - f_b(N_{Ah})]\,. \tag{A.6}$$

Now again from strict concavity of $f_a - f_b$, one has with $N_{Ah} > n_{ah}$ and $n_a = n_{ah} + n_{a\ell}$ that $f_a(n_a) - f_b(n_a) - [f_a(n_{ah}) - f_b(n_{ah})] > f_a(N_{Ah} + n_{a\ell}) - f_b(N_{Ah} + n_{a\ell}) - [f_a(N_{Ah}) - f_b(N_{Ah})]$. Hence, (A.6) is satisfied if

$$\begin{aligned} & f_a(N_{Ah} + n_{a\ell}) - f_b(N_{Ah} + n_{a\ell}) - [f_a(N_{Ah}) - f_b(N_{Ah})] \\ > \;& \rho_\ell[f_a(N_A) - f_b(N_A)] - \rho_\ell[f_a(N_{Ah}) - f_b(N_{Ah})]\,. \end{aligned} \tag{A.7}$$

Since $N_{Ah} + n_{a\ell} = \rho_\ell N_A + (1 - \rho_\ell) N_{Ah}$, inequality (A.7) is equivalent to

$$\begin{aligned} & f_a\Big(\rho_\ell N_A + (1-\rho_\ell)N_{Ah}\Big) - f_b\Big(\rho_\ell N_A + (1-\rho_\ell)N_{Ah}\Big) \\ > \;& \rho_\ell[f_a(N_A) - f_b(N_A)] + (1-\rho_\ell)[f_a(N_{Ah}) - f_b(N_{Ah})]\,, \end{aligned}$$

which is true by the strict concavity of $f_a - f_b$. ∎

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